Korean J. of Chem. Eng., 11(4), 246-253 (1994)

A COLLISION MODEL FOR FINE PARTICLES IN A TURBULENT SYSTEM

Yoon-Bong Hahn

Department of Chemical Engineering, Chonbuk National University, Chonju 560-756, Korea
(Received 26 November 1993 • accepted 29 June 1994)

Abstract—A model is proposed to describe the collision rate of small particles suspended in a turbulent system. The model combines the possible collision mechanisms: 1) collisions due to the relative velocity between fluid and particles, and 2) collisions due to the turbulent diffusion of particles. This model also accounts for the effect of particle concentration on the collision rate. It was found that the turbulent diffusion of particles plays an important role in the collision of equally sized particles as well as of unequally sized particles. The model predictions also show that the collision rate of particles is strongly affected by the concentration of solid particles and by the turbulence intensity. Much more reliable predictions than previously possible have been obtained with the present model.

INTRODUCTION

Particle-laden turbulent fluid systems occur in many engineering applications. These particulate systems involve collisions of particles. Understanding of the collision process of small particles suspended in a turbulent system has been an important issue to many chemical and mechanical engineers. In spite of the increased efforts to understand the collision process, the complete description of the collision rate of small particles remains unresolved. This is mainly due to the lack of basic understanding of the collision mechanisms.

The collision mechanisms of small particles suspended in a turbulent system may be described as:

(1) collisions due to the relative velocity between fluid and particles and due to the spatial variation of fluid velocity surrounding two colliding particles, and

(2) collisions due to the turbulent diffusion of particles.

The first mechanism is important for the collision of unequally sized particles, while the second mechanism is significant for particles of equal size as well as unequal size.

Several investigators have studied the collision process of small particles suspended in turbulent systems. Saffman and Turner [1] studied the collision rate of drops in turbulent clouds by considering that collisions occurred due to spatial variation of the fluid velocity, and also due to different velocities of particles of unequal size. Yuu [2] discussed the collision rate of small particles in a homogeneous and isotropic turbulence. He confirmed that the relative velocity between fluid and particles was the main factor in turbulent coagulation of unequally sized particles. He also mentioned that the most important factors affecting the turbulent coagulation were the dissipation rate of turbulent kinetic energy, and the particle relaxation time. His conclusions were, however, similar to those mentioned by Saffman and Turner. Beal [3] studied the agglomeration of particles in a turbulent flow. He assumed that collisions were the result of particle movement caused by a combination of the Brownian motion of particles and the turbulent diffusion of particles. He predicted the turbulent agglomeration of suspended particles with the diffusion model, which was originally proposed by Levich [4]. However, he did not account for collisions due to the relative velocity between fluid and particles.

The investigators mentioned above did not account for all the possible mechanisms. Futhermore, their models did not include the effect of particle concentration on the collision rate.

The purpose of this work is to propose a collision model which will adequately predict the collision rate of small particles suspended in a turbulent system. The proposed model combines the possible collision mechanisms mentioned above. This model also includes the effect of the concentration of particles on the collision rate. The results obtained by the present model will finally be compared with the previously obtained results.

MATHEMATICAL MODEL

In order to adequately describe the collision process of small particles, the possible collision mechanisms mentioned above have to be considered simultaneously because they do not occur separately. The collision rate of small particles may then be expressed as:

[Overall collision rate]=
[Collision rate due to the relative velocity between fluid and small particles, and due to the spatial variation of fluid velocity]+[Collision rate due to the turbulent diffusion of particles] (1)

Eq. (1) is the principal model in this work for the collision rate of small particles suspended in a turbulent system.

The following assumptions are used as a basis for the model equations:

(1) The turbulence is homogeneous and isotropic.

(2) The particles are smaller than the small eddies of the turbulence, and the particle relaxation time is small compared with the time scale of the smallest eddies.

(3) The particles are of spherical shape and the drag on each particle is determined by Stoke's Law.

(4) The probability distribution function of the relative velocity of two particles may be expressed in Gaussian form [1, 2].

Within these assumptions, the proposed model of the turbulent collision of fine particles can be simply expressed as:

$$N_t = N_c + N_d \tag{2}$$

where N_t represents the term on the left-hand side of Eq. (1), and N_c and N_d represent the first and second terms on the right-hand side of Eq. (1), respectively.

Consider two particles as shown in Fig. 1. The vectors $\vec{V}_{p1}$ and $\vec{V}_{p2}$ represent particle velocities of radii r_{p1} and r_{p2}, respectively. The vectors $\vec{U}_1$ and $\vec{U}_2$ represent velocities of the fluid surrounding the two particles of sizes r_{p1} and r_{p2}, respectively.

1. Collision Rate due to Relative Velocity and Spatial Variation of Fluid Velocity, N_c

When particles are involved in a fluid flow, the fluid is distorted due to the presence of particles and the motion of a particle is affected by the motion of neighboring particles. Since the trajectory of a particle is affected by the distored stream of fluid and the motion

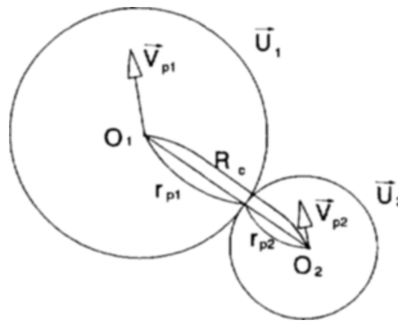

Fig. 1. Collision of two particles.

of neighboring particles, the particles tend to be pushed away from each other and two particles approaching each other do not always collide. Hence the collision efficiency, which is a measure of the extent of collision, has to be taken into account. Without considering collision due to turbulent diffusion of particles, N_c may be described as:

$$N_c = \pi\eta R_c^2\, n_1 n_2 \iiint_{-\infty}^{\infty} |\vec{W}| P(\vec{W})\, d\vec{W} \tag{3}$$

where $R_c = r_{p1} + r_{p2}$.

In Eq. (3), η is the collision efficiency; n_1 and n_2 are the number concentrations of particles of radii r_{p1} and r_{p2}, respectively; $|\vec{W}| = |\vec{V}_{p1} - \vec{V}_{p2}|$ is the magnitude of the relative velocity vector of two colliding particles; $P(\vec{W})$=the probability distribution of the relative velocity $\vec{W}$.

In a homogeneous and isotropic turbulence, $P(\vec{W})$ may be expressed in a Gaussian form [1, 2]:

$$P(\vec{W}) = \left(\frac{3}{2\pi\ \mathrm{var}(\vec{W})}\right)^{3/2} \exp\left(-\frac{3}{2}\frac{|\vec{W}|^2}{\mathrm{var}|\vec{W}|}\right) \tag{4}$$

Substituting Eq. (4) into Eq. (3) and integrating Eq. (3) yield

$$N_c = \left(\frac{8\pi}{3}\right)^{1/2} \eta R_c^2\ n_1 n_2 [\mathrm{var}(\vec{W})]^{1/2} \tag{5}$$

using the transformations of $W_x = |\vec{W}|\cos\theta\cos\psi$, $W_y = |\vec{W}|\cos\theta\sin\psi$ and $W_z = |\vec{W}|\sin\theta$.

The collision rate of particles due to the relative velocity between the fluid and particles can then be obtained by Eq. (5) when the variance of the relative velocity of particles is known. The variance of $\vec{W}$ can be expressed as:

$$\mathrm{var}(\vec{W}) = \mathrm{var}(\vec{S}_1 - \vec{S}_2) + \mathrm{var}(\vec{U}_1 - \vec{U}_2) \tag{6}$$

where

$$\vec{S}_1=\vec{V}_{p1}-\vec{U}_1 \tag{7}$$

$$\vec{S}_2=\vec{V}_{p2}-\vec{U}_2 \tag{8}$$

In Eq. (6), it is assumed that $(\vec{U}_1-\vec{U}_2)$ is statistically independent of $\vec{S}_1$ and $\vec{S}_2$.

From the assumption of the isotropy of small eddies, the variance of relative velocity of fluid elements surrounding two particles can be approximated as [1]:

$$\mathrm{var}(\vec{U}_1-\vec{U}_2)=\frac{R_c^2}{3}\frac{\varepsilon}{\nu_f} \tag{9}$$

To calculate the first term on the right-hand side of Eq. (6), we can use a particle momentum equation for a single particle in the Lagrangian form:

$$\frac{d\vec{V}_p}{dt}=\frac{\rho_f}{\rho_p}\frac{d\vec{U}}{dt}+\frac{1}{\tau}(\vec{U}-\vec{V}_p)+\left(\frac{\rho_f}{\rho_p}-1\right)\vec{g} \tag{10}$$

The term on the left-hand side denotes the acceleration of the particle. The first term on the right-hand side represents the force from pressure gradient in the fluid surrounding the particle. The second term is the viscous resistance force according to Stoke's law. The third term represents the body force acting on the particle, such as gravity force.

The particle relaxation time τ is defined as:

$$\tau=\frac{2r_p^2}{9\nu_f}\left(\frac{\rho_f}{\rho_p}\right) \tag{11}$$

by assuming that small spherical particles obey Stoke's law. Using Eqs. (7) and (8), Eq. (10) can be rewritten as:

$$\frac{d\vec{S}}{dt}=\left(\frac{\rho_f}{\rho_p}-1\right)\frac{d\vec{U}}{dt}-\frac{\vec{S}}{\tau}+\left(\frac{\rho_f}{\rho_p}-1\right)\vec{g} \tag{12}$$

With the assumption that the particle is smaller than the small eddies of turbulence or τ is smaller than the time scale of smallest eddies, the variation of $\vec{S}$ with respect to time can then be assumed to be negligible, and Eq. (12) is reduced to:

$$\vec{S}=\tau\left(\frac{\rho_f}{\rho_p}-1\right)\left(\frac{d\vec{U}}{dt}+\vec{g}\right) \tag{13}$$

Taking the time average of $\vec{S}^2$ for a particle size of r_{p1}, and assuming no relationship between $(d\vec{U}/dt)$ and $\vec{g}$, we get

$$\overline{\vec{S}_1^2}=\tau_1^2\left(1-\frac{\rho_f}{\rho_p}\right)^2\left\{\overline{\left(\frac{d\vec{U}}{dt}\right)^2}+\vec{g}^2\right\} \tag{14}$$

where the over bar denotes "time averaged quantity".

Since in the Lagrangian framework, the fluid velocity can be expressed as a function of time and space, taking a derivative with respect to time gives

$$\frac{d\vec{U}}{dt}=\frac{D\vec{U}}{Dt} \tag{15}$$

where D/Dt denote the substantial derivative. Using Eq. (15) and the assumption of isotropic turbulence, we get

$$\overline{\left(\frac{d\vec{U}}{dt}\right)^2}=\overline{\left(\frac{D\vec{U}}{Dt}\right)^2}=3\overline{\left(\frac{Du}{Dt}\right)^2} \tag{16}$$

where u denotes the root mean square turbulence velocity of fluid. Then Eq. (14) can be rewritten as:

$$\overline{\vec{S}_1^2}=\tau_1^2\left(1-\frac{\rho_f}{\rho_p}\right)^2\left\{3\overline{\left(\frac{Du}{Dt}\right)^2}+g^2\right\} \tag{17}$$

Since a similar equation to Eq. (17) is obtained for particle of size r_{p2}, the variance of the relative velocity between the fluid and particles can then be expressed as:

$$\mathrm{var}(\vec{S}_1-\vec{S}_2)=(\tau_1-\tau_2)^2\left(1-\frac{\rho_f}{\rho_p}\right)^2\left\{3\overline{\left(\frac{Du}{Dt}\right)^2}+g^2\right\} \tag{18}$$

Using Eqs. (9) and (18), the variance of relative velocity between two colliding particles can then be given as:

$$\mathrm{var}(\vec{W})=(\tau_1-\tau_2)^2\left(1-\frac{\rho_f}{\rho_p}\right)^2\left\{3\overline{\left(\frac{Du}{Dt}\right)^2}+g^2\right\}+\frac{1}{3}R_c^2\frac{\varepsilon}{\nu_f} \tag{19}$$

Substituting Eq. (19) into Eq. (5) yields the collision rate due to the relative velocity between the fluid and particles, and due to the spatial variation of fluid velociy.

$$N_c=\left(\frac{8\pi}{3}\right)^{1/2}\eta R_c^2 n_1 n_2\left[(\tau_1-\tau_2)^2\left(1-\frac{\rho_f}{\rho_p}\right)^2\left\{3\overline{\left(\frac{Du}{Dt}\right)^2}+g^2\right\}+\frac{1}{3}R_c^2\frac{\varepsilon}{\nu_f}\right]^{1/2} \tag{20}$$

Eq. (20) is similar to the equations obtained by Saffman and Turner [1] and Yuu [2]. The coagulation coefficient due to the collision caused by the variance of the relative velocity between the fluid and partices can then be defined as:

$$K_c=\frac{N_c}{\eta n_1 n_2} \tag{21}$$

or

$$K_c=\left(\frac{8\pi}{3}\right)^{1/2}R_c^2\left[(\tau_1-\tau_2)^2\left(1-\frac{\rho_f}{\rho_p}\right)^2\left\{3\overline{\left(\frac{Du}{Dt}\right)^2}+g^2\right\}\right.$$

$$+\frac{1}{3}R_c^2\frac{\varepsilon}{\nu_f}\Big]^{1/2} \tag{22}$$

2. Collision Rate due to Turbulent Diffusion of Particles, N_d

As mentioned previously, the diffusion of particles suspended in a turbulent flow can also cause particles to be collided. We may assume that the particles are transferred by isotropic turbulence, provided that the size of particle is smaller than the microscale of turbulence. Since the particles contained in turbulent eddies migrate through the bulk fluid in a chaotic fashion, the eddy motion of particles may be described by a certain turbulent diffusion coefficient.

Because of the complexity of turbulence, it is assumed that one particle is stationary and other particles are diffusing toward it [3, 4]. The turbulent diffusion equation of particles is then expressed as:

$$\frac{\partial n_p}{\partial t}=\vec{\nabla}\Gamma_d\ \vec{\nabla}n_p \tag{23}$$

where Γ_d denotes the turbulent diffusivity of particles and n_p represents the number of particles per unit volume.

At steady state, Eq. (23) can be rewritten as in spherical coordinates considering the radial direction only;

$$\frac{1}{r^2}\frac{d}{dr}\left(\Gamma_d r^2\frac{dn_p}{dr}\right)=0 \tag{24}$$

with boundary conditions:

$$n_p=0 \quad \text{at } r=R_c \tag{25a}$$

$$n_p=n_o \quad \text{at } r\rightarrow\infty \tag{25b}$$

where n_o is the averaged uniform concentration of diffusing particles a great distance away from the stationary particle.

Since the turbulent diffusivity is a function of the eddy scale and varies from point to point, Γ_d is a variable quantity and is given by Levich [4]:

$$\Gamma_d=\beta(\varepsilon/\nu_f)^{1/2}\ r^2 \quad \text{for } R_c<\lambda_o \tag{26}$$

where λ_o is the micro-scale of turbulence, and β is a constant.

By integrating Eq. (24) with Eq. (26), using boundary conditions (25a) and (25b), and allowing for continuity of n_p at $r=\lambda_o$, we can get the flux of particles across the surface of the sphere of collision $r=R_c$:

$$J=\left(\Gamma_d\frac{dn_p}{dr}\right)_{r=R_c} \tag{27}$$

Then, the obtained equation is:

$$J=\frac{3\ n_oR_ca}{1+\frac{2}{7}\left(\frac{R_c}{\lambda_o}\right)^3} \quad \text{for } R_c\leq\lambda_o \tag{28}$$

where

$$a=\beta\sqrt{\varepsilon/\nu_f} \tag{29}$$

and $\beta=1/\sqrt{15}$, which are given by Beal [3] and Fuchs [5].

The micro-scale of turbulence is given by Levich [4] as:

$$\lambda_o=\left(\frac{\nu_f^3}{\varepsilon}\right)^{1/4} \tag{30}$$

The collision rates due to the turbulent diffusion are then obtained by multiplying Eq. (28) with $4\pi R_c^2$ so that

$$N_d=\frac{12\pi n_oR_c^3a}{1+\frac{2}{7}\left(\frac{R_c}{\lambda_o}\right)^3} \quad \text{for } R_c\leq\lambda_o \tag{31}$$

By normalizing the collision rates with n_o, we can express the coagulation coefficient due to the particle diffusion as:

$$K_d=\frac{12\pi R_c^3a}{1+\frac{2}{7}\left(\frac{R_c}{\lambda_o}\right)^3} \quad \text{for } R_c\leq\lambda_o \tag{32}$$

Eqs. (31) and (32) are similar to ones obtained by Levich and Beal when the Brownian diffusion of particles is neglected.

3. Overall Collision Rate of Small Particles in Turbulence, N_t

The collision rate of small particles in a turbulent flow, defined in Eq. (2), can then be obtained from Eqs. (20) and (31). The corresponding overall coagulation coefficient, K_t, can be expressed as:

$$K_t=K_c+K_d \tag{33}$$

When the particles follow the turbulent flow completely and the particle relaxation times become zero, or when $r_{p1}=r_{p2}$ or $\rho_p=\rho_f$, the collision rate N_c is reduced to

$$N_c=\left(\frac{8\pi}{9}\right)^{1/2}\eta R_c^3n_1n_2\left(\frac{\varepsilon}{\nu_f}\right)^{1/2} \tag{34}$$

The corresponding coagulation coefficient K_c is then obtained as:

$$K_r=\left(\frac{8\pi}{9}\right)^{1/2}R_r^3\left(\frac{\varepsilon}{\nu_f}\right)^{1/2}=1.671R_r^3\left(\frac{\varepsilon}{\nu_f}\right)^{1/2} \quad (35)$$

Hence, the coagulation coefficient of small particles of equal size can be expressed as:

$$K_t=1.671R_r^3\left(\frac{\varepsilon}{\nu_f}\right)^{1/2}+\frac{12\pi R_r^3 a}{1+\frac{2}{7}\left(\frac{R_r}{\lambda_o}\right)^3} \quad \text{for } R_r\leq\lambda_o \quad (36)$$

Eq. (36) showes that when the particles are of equal size the collision or coagulation of particles in a turbulent flow is mainly due to the spatial variation of fluid velocity surrounding the particles and the turbulent diffusion of particles.

4. Kinematic Viscosity of a Particle-Laden Turbulent Flow

When particles are introduced into a turbulent flow, the flow field is affected by the presence of particles. Owen [6] argued that in a shear flow the presence of particles would lead to an increase in the energy dissipation in the ratio of $(1+\overline{\rho}_b/\rho_f)$ when the particle relaxation time (τ) was much less than the turbulence time scale (t^*). He also mentioned that the quantity controlling the interaction between fluid and particles was the particle relaxation time.

The kinematic viscosity of the fluid in a two phase flow can then be expressed with Owen's correlation equation (also see Melville and Bray [7]):

$$\nu_f=\nu_f^o\left(1+\frac{\overline{\rho}_b}{\rho_f}\right)^{-1/2} \quad \text{for } \tau\ll t^* \quad (37)$$

and

$$\nu_f=\nu_f^o\left(1+\frac{\overline{\rho}_b t^*}{\rho_f \tau}\right)^{-1/2} \quad \text{for } \tau\geq t^* \quad (38)$$

where ν_f^o is the kinematic viscosity of clean fluid and $\overline{\rho}_b$ represents the mean mass of particles per unit volume of the mixture. $\overline{\rho}_b$ can be obtained by the equation:

$$\overline{\rho}_b=\rho_p\theta_v \quad (39)$$

where ρ_p is the mass density of particle itself and θ_v is the volume fraction of particles in a unit volume of the mixture. It is noted that Eqs. (37) and (38) are used in a dilute system, i.e. when $\theta_v\ll 1$.

Since it was assumed that the particle relaxtion time is small compared to the time scale of the smallest eddies, Eq. (37) was used to calculate the kinematic viscosity of the fluid throughout this work. The concentration of particles or the volume fraction of particles(θ_v) affects the kinematic eddy viscosity ν_f according to Eqs. (37) and (38), and consequently the collision rate of small particles is affected by θ_v. It is also noted that the increased concentration of particles may drive more particle-particle interaction and then affect the collision rate.

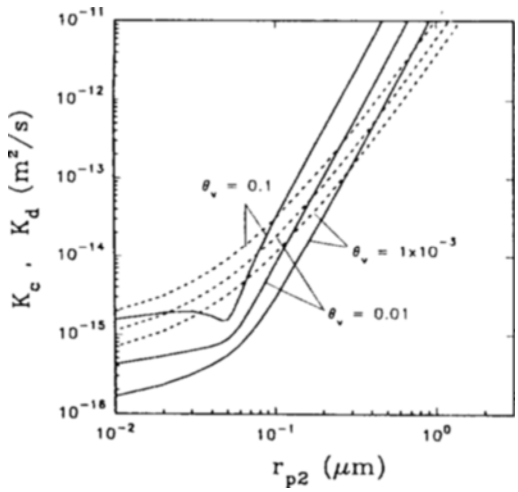

Fig. 2. Effect of particle concentration (θ_v) on coagulation coefficients K_c and K_d ($r_{p1}=0.05$ μm and $\varepsilon=100$ m^2/s^3): — K_c and --- K_d.

RESULTS AND DISCUSSION

The overall collision rate of small particles suspended in a turbulent system can be estimated by combining Eqs. (20) and (31). For the purpose of comparision of collision models, the same conditions as ones used by Yuu [2] were used for calculations, i.e., air at ambient temperature, $\rho_p=2{,}000$ kg/m^3 and $\varepsilon=100$ m^2/s^3.

Coagulation coefficients K_c and K_d defined in Eqs. (22) and (32), respectively, were plotted against r_{p2} with varying θ_v as a parameter in Figs. 2 and 3 in which values of r_{p1} were given as 0.05 and 0.5 μm, respectively. These figures show that for smaller particles, the coagulation coefficient K_d due to the turbulent diffusion is greater than K_c (Fig. 2), and vice versa for larger particles (Fig. 3). On the other hand, the coagulation coefficient K_c due to the relative velocities is greater for larger particles than for smaller particles. These results indicate that for smaller particles, the collision due to the turbulent diffusion may play a more important role than that due to the relative velocity; while for larger particles, the relative velocity between the fluid and particles may be a more important factor for collision than the turbulent diffusion. These phenomena may be explained by the fact that

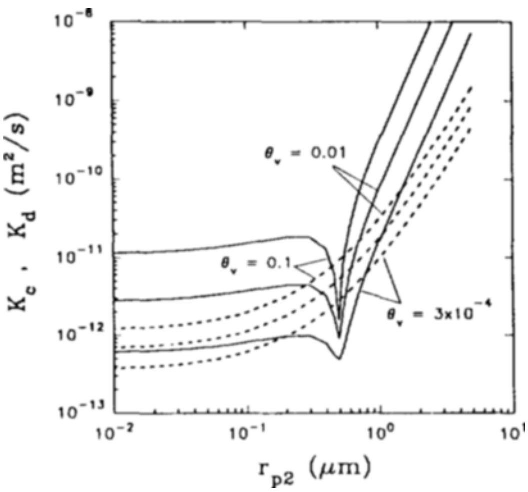

Fig. 3. Effect of particle concentration (θ_v) on coagulation coefficients K_c and K_d ($r_{p1}=0.5$ μm and $\varepsilon=100$ m^2/s^3): — K_c and --- K_d.

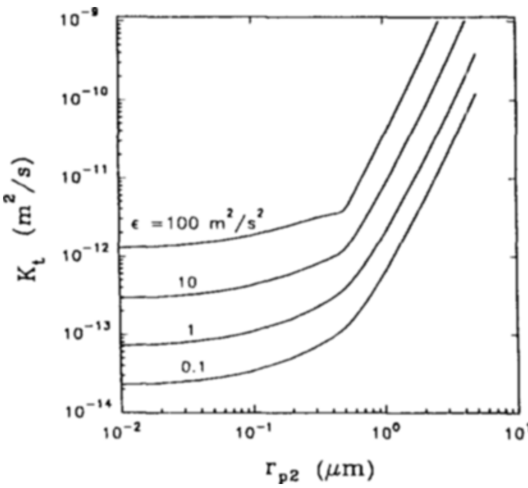

Fig. 4. Effect of particle concentration (θ_v) on the overall coagulation coefficient K_t ($r_{p1}=0.5$ μm and $\varepsilon=100$ m^2/s^3).

smaller particles more closely follow the turbulent fluctuation compared with larger particles. hence the turbulent diffusion of the smaller particles could occur more easily and force the particles to be collided. For larger particles, which do not closely follow the turbulent fluctuation, more collisions could occur due to the relative velocities rather than due to the turbulent diffusion of particles. Hence, it is concluded that the collision of particles either due to the turbulent diffusion or due to the relative velocity can not be neglected quantitatively in a certain range of particle size because the difference of order of magnitudes of K_c and K_d is not large.

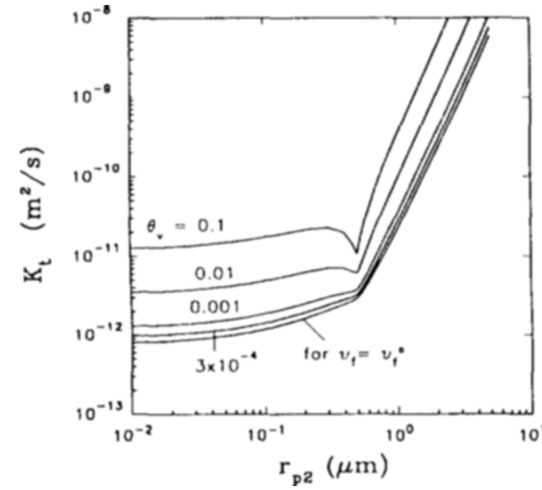

Fig. 5. Effect of the dissipation rate of turbulent kinetic energy on the overall coagulation coefficient K_t (r_{p1} $=0.5$ μm and $\theta_v=1\times10^{-3}$).

The figures also show that coagulation coefficients increase as the particle concentration increases. This result is not surprising, since it can be explained by the fact that although the dissipation rate of turbulent kinetic energy may be affected by the increased particle concentration, the particle-laden turbulent flow becomes more chaotic, and particles have a greater chance of collision as the particle concentration increases.

Fig. 4 shows the plot of the overall coagulation coefficient (K_t) against r_{p2} for a given $r_{p1}=0.5$ μm with varying θ_v as a parameter. As expected, K_t increases as θ_v increases. The figure also shows the calculated results for $\nu_t=\nu_t^*$. It is noted that if ν_t is not corrected by Eq. (37) as Saffman/Turner and Yuu did, the coagulation coefficient or the collision rate is independent of the concenration of particles. In realistic systems, however, the collision rate of particles is affected by the concentration of particles. Hence it follows that the proposed model in this work, which takes account of particle concentration and all the possible collision mechanisms, gives more reliable predictions than those of previous work.

Fig. 5 shows the effect of the dissipation rate of turbulent kinetic energy on collision. Since in general a strong turbulence causes the particle-laden flow to be more turbulent and chaotic, particles have more chance to collide and the collision rate or the coagula-

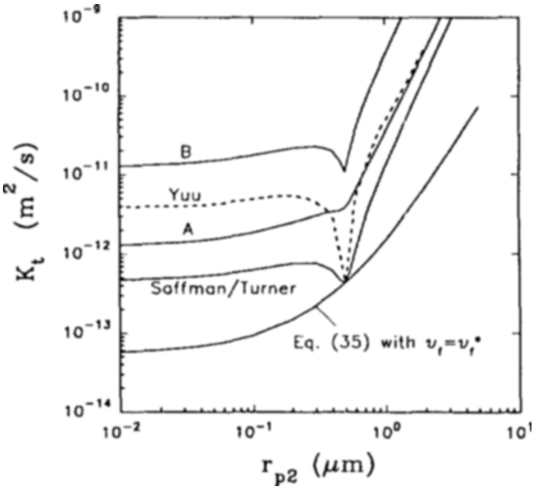

Fig. 6. Comparison of collision models by Saffman/Turner [1], Yuu [2] and this work ($r_{p1}=0.5$ μm and $\varepsilon=100$ m^2/s^3). Predictions by this work: A ($\theta_v=1\times10^{-3}$) and B ($\theta_v=0.1$).

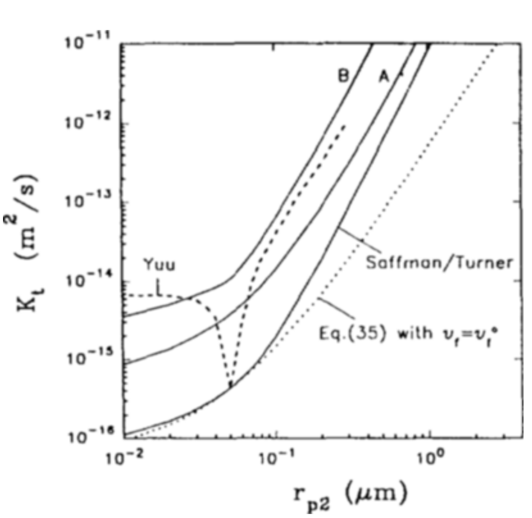

Fig. 7. Comparision of collision models by Saffman/Turner [1], Yuu [2] and this work ($r_{p1}=0.05$ μm and $\varepsilon=100$ m^2/s^3). Predictions by this work: A ($\theta_v=1\times10^{-3}$) and B ($\theta_v=0.1$).

tion coefficient increases as ε increases.

Comparisions of predictions obtained by the proposed model with those by Saffman/Turner and Yuu are shown in Figs. 6 and 7. According to their predictions, the curves show sharp valleys near the point of $r_{p1}=r_{p2}$ and have minimum values of K_t at $r_{p1}=r_{p2}$ (Fig. 6). The reason for these results is that according to their models, when $r_{p1}=r_{p2}$ particles are not collided due to the relative velocity between particles, then collisions occur only due to the weaker factor of the relative velocity between the fluid elements surrounding the particles. This is also because they did not take account of the collisions due to the turbulent diffusion of particles. The prediction made by the present model shows no such valley for $\theta_v=1\times10^{-3}$ (curve A), but it shows a similar result for higher θ_v (curve B) to those results by Yuu, and Saffman and Turner. Hence, it follows that the collision rate is also strongly affected by the concentration of particles. The curve obtained by Eq. (35) with $v_f=v_f^o$ shows the lowest values of K_t, hence Eq. (35) can not be used for adequately predicting the collision rate even for small particles but can be used only to get rough predictions. It is also seen by comparing magnitudes of K_t from Figs. 6 and 7 that the collision or coagulation coefficient decreases with decreasing particle radii. Although prediction by Yuu shows a sharp valley near the point $r_{p1}=r_{p2}$, the predictions made by the present work do not (Fig. 7).

For the turbulent diffusion of particles, it was assumed that the particles stick together upon collision. However, this assumption may not be actually true because the inter-particle forces may not only prevent particles from sticking together but also prevent their collision. In order to precisely predict the collision process due to the turbulent diffusion, a more advanced model may then be required to include such a inter-particle force, and consequently the model will be of more complicated form.

Within this limitation, however, it may be concluded that the reliability of prediction obtained by the present model represents a substantial improvement over those of previous work.

CONCLUSION

A model was derived to describe the collision process of small particles suspended in a turbulent system. The model includes the collision due to the relative velocities and the collision due to the turbulent diffusion of partcles. The predictions by the proposed model showed that for small particles, the collision due to the diffusion of particles is greater than that due to the relative velocities, and vice versa for larger particles. It was also found that the collision rate of particles are strongly affected by the concentration of particles and the turbulent intensity. In spite of some limitations of the model, much more reliable predictions than previously possible have been obtained with the present model.

ACKNOWLEDGEMENT

This work was supported in part by NON DIRECTED RESEARCH FUND, Korea Research Foundation, 1993.

NOMENCLATURE

a : parameter defined in Eq. (29)
g : the gravity force
J : the flux of colliding particles [# of particles/m^2·s]
K_c : the coagulation coefficient defined in Eq. (21) [m^2/s]
K_d : the coagulation coefficient defined in Eq. (32) [m^2/s]
K_t : the overall coagulation coefficient defined in Eq. (33) [m^2/s]
N_c : the collision rate defined in Eq. (20) [$m^{-3}s^{-1}$]
N_d : the collision rate defined in Eq. (31) [$m^{-3}s^{-1}$]
N_t : the total collision rate defined in Eq.(2) [$m^{-3}s^{-1}$]
n_1, n_2 : number concentrations of particles of size r_{p1} and r_{p2}, respectively [m^{-3}]
n_o : the averaged uniform number concentration of particle of any size [m^{-3}]
R_c : the collision distance [m]
r : radial coordinate
r_{p1}, r_{p2} : particle radii of particle 1 and 2, respectively [m]
$\vec{S}$: the vector of relative velocity between the fluid and particles, Eqs. (7) and (8)
t : time [s]
t* : the turbulence time scale [s]
$\vec{U}_1$, $\vec{U}_2$: velocity vectors of the fluid elements surrounding particles of sizes of r_{p1} and r_{p2}, respectively [m/s]
u : the root mean square turbulence velocity [m/s]
$\vec{V}_{p1}$, $\vec{V}_{p2}$: velocity vectors of particles of size r_{p1} and r_{p2}, respectively [m/s]
$\vec{W}$: the relative velocity vector of two colliding particles [m/s]

Greek Letters

ε : dissipation rate of turbulent kinetic energy [m^2/s^3]
Γ_d : particle diffusivity [m^2/s]
θ_v : the volume fraction of particles [-]
λ_o : the micro-scale of turbulence [m]
ν_f : the kinematic viscosity of the particle-laden fluid [m^2/s]
ν_f^o : the kinematic viscosity of the clean fluid [m^2/s]
ρ_f : the density of fluid [kg/m^3]
ρ_p : the mass density of particle [kg/m^3]
$\overline{\rho_b}$: the mean mass of particles per unit volume of the mixture [kg/m^3]
τ : the particle relaxtion time defined in Eq. (11)
η : collision efficiency

REFERENCES

1. Saffman, P. G. and Turner, J. S.: *J. Fluid Mech.*, **1**, 16 (1956).
2. Yuu, S.: *AIChE J.*, **30**, 802 (1984).
3. Beal, S. K.: *J. Aerosol Science*, **3**, 113 (1972).
4. Levich, V. G.: "Physicochemical Hydrodynamics", Prentice-Hall, Englewood Cliffs, New Jersey, 213 (1962).
5. Fuchs, N. A.: "The Mechanics of Aerosols", Pergamon Press, Oxford, 299 (1964).
6. Owen, P. R.: *J. Fluid Mech.*, **39**, 407 (1969).
7. Melville, W. K. and Bray, K. N. C.: *Int. J. Heat Mass Transfer*, **22**, 647 (1979).
8. Sastry, K. V. S. and Gaschiguard. P.: *Ind. Eng. Chem. Fundam.*, **20**, 355 (1981).